HAREZMI EDUCATION MODEL IN PRIMARY MATHEMATICS EFFECTS ON PROBLEM-SOLVING AND CRITICAL THINKING

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Abstract

Interdisciplinary approaches in education play a crucial role in developing students' real-world mathematical problem-solving (MPS) skills and critical thinking (CT) skills. One such approach, the Harezmi Education Model (HEM), integrates mathematics, science, computer technologies, social sciences, the arts, and sports through data-driven, technology-supported learning environments. This research sought to explore how the HEM can be incorporated into primary mathematics teaching and to determine its effect on learners' MPS and CT skills. The study was designed according to an explanatory sequential mixed-methods approach. During the quantitative stage, a quasi-experimental pretest–posttest model was implemented with fourth-grade students, who were separated into an experimental group (n = 26) and a control group (n = 26). In the subsequent qualitative stage, semi-structured interviews were carried out with 15 primary school teachers. The quantitative data were examined through multivariate analysis of variance (MANOVA), whereas the qualitative data were interpreted using content analysis. The results demonstrated that instruction grounded in HEM produced statistically significant improvements in students' MPS and CT skills. Teacher perspectives further indicated that HEM supports interdisciplinary learning and real-world connections, although challenges related to resources, time management, and teacher preparedness were noted. Overall, the results suggest that HEM offers a promising interdisciplinary framework for enhancing mathematics instruction and 21st-century skills in primary education.

Keywords: Harezmi education model, mathematical problem-solving, critical thinking, interdisciplinary learning, primary education.

INTRODUCTION

Interdisciplinary approaches in education are critical for developing students' real-world problem-solving abilities and supporting their CT skills (Tokmak et al. 2023). One such approach, the HEM, is an innovative educational model that brings together computer technologies, science and mathematics education, social sciences, art and design, and sports sciences, while maintaining interdisciplinary integrity. This model enables students to identify real-world problems grounded in data, develop innovative solutions, and implement these solutions in technology-supported learning environments (Çimşir et al. 2022). Additionally, the model provides opportunities for teachers to develop their interdisciplinary and collaborative skills, integrate technology effectively, and create research- and development-oriented learning designs (Çimşir, 2024; Koçoğlu, 2018). Additionally, it encourages students to engage with technological learning tools—such as programming, robotics, and game design—in order to enhance their algorithmic thinking and problem-solving abilities (Çimşir, 2024).

This study investigates the impact of HEM-based problem-solving instruction on students and examines the implementation of the HEM in primary school mathematics classes through the experiences and perspectives of primary school teachers. With its interdisciplinary structure, HEM offers a framework for developing students' 21st-century skills, similar to the internationally

widespread STEM (Science, Technology, Engineering, Mathematics) and STEAM (STEM with added Arts) approaches. HEM adds a unique dimension to STEM/STEAM approaches, with a structure focused on coding, algorithmic thinking, computer science activities, and collaborative product development. As a model developed specifically within a local context, it reinterprets and contextualizes these global approaches from cultural and pedagogical perspectives. In this regard, HEM stands out for its distinctiveness among interdisciplinary teaching models implemented in Türkiye. It contributes to the international literature by presenting a different view on STEM/STEAM education. The results of this study reveal HEM's contributions to mathematics education and areas for improvement, which are expected to enhance the quality of the model at the national level and to offer an alternative model for interdisciplinary education approaches at the international level.

Interdisciplinary Approach and the HEM

An interdisciplinary approach is an instructional strategy that fosters deep and multidimensional learning by integrating the knowledge, skills, and viewpoints of multiple fields around a common theme or problem (Aydın & Balım, 2005). Through this approach, learners can recognize connections across different subject areas, transfer their learning to real-world contexts, and examine authentic problems holistically (Martinello & Cook, 2000). Thanks to this method, learners not only gain academic knowledge but also have the opportunity to develop higher-level skills such as problem solving, creativity, analytical thinking, and collaboration (Keskin et al., 2024; Erdem & Eminoğlu-Küçüktepe, 2024). Teaching abstract subjects such as mathematics and science by relating them to daily life increases students' interest, makes learning more lasting, and positively affects academic achievement (Demirel et al., 2008; Karakuş et al., 2017). Learning processes conducted with an interdisciplinary approach encourage students' active participation, foster their self-confidence, and promote positive attitudes toward school and lessons (Güder & Gürbüz, 2018; Gürkan, 2019). In this context, the contemporary understanding of education aims to develop students' 21st-century skills and equip them with competencies such as interpreting information, problem solving, making inferences, and CT (Uğraş, 2017).

HEM is an innovative model in which students identify real-world problems in a data-driven manner, develop creative, solution-oriented approaches to these problems, and teachers manage the process from an interdisciplinary perspective (Koçoğlu, 2018). The fundamental elements of the model are interdisciplinary integration, real-world problems, and the nature of the technological tools used to solve them (Ayra et al., 2025; Babayiğit-Durakcan, 2024). This model aims to integrate different disciplines and embed computational thinking into daily life (Ceylan et al., 2020; Koçoğlu, 2018). Furthermore, one of the fundamental steps of the model is for students to generate solutions through algorithmic thinking based on problems they identify (Çimşir, 2024). Developed in line with global education reforms, HEM aligns with approaches such as STEM and STEAM. Still, it stands out as an alternative model in Türkiye for its focus on interdisciplinary teacher cooperation, ethics, and safety, as well as its continuously updated structure.

Primary School Mathematics Education and MPS

Primary school mathematics education is important because it coincides with a critical period in which the foundations of individuals' cognitive development are established. Mathematics education during this period should not be limited to teaching basic operations; it should also support the development of students' higher-level skills such as reasoning, making connections, predicting, problem-solving, and communication (Filiz & Güneş, 2023). Due to its abstract nature, mathematics can be challenging for students. Therefore, teachers need to concretize mathematical concepts by relating them to students' daily lives (Orhan, 2025). In this process, students' understanding of the problems they encounter, their ability to generate solutions, and their development of alternative perspectives directly contribute to the development of their MPS skills (Çimşir & Baysal, 2019). At this point, instead of directly transferring knowledge to students, students should be supported in learning how to access information, and the teaching process should be enriched with structured, thinking-based activities (Aydoğdu & Kesercioğlu, 2005).

Effective mathematics instruction in the early years should enable students to express their thoughts through mathematical representations, thereby enhancing both the learning process and student engagement (Büyükalan Filiz & Ergan, 2020). This is possible when students can establish connections between abstract symbols and the concrete world, focus on mathematical concepts, and discover relationships among concepts (Ulu et al., 2016). In this context, teachers should not reduce mathematics teaching to technical skills alone. Instead, they should adopt an approach that aims to develop CT, reasoning, and analytical thinking skills, enabling students to understand the problems they encounter in daily life and develop practical solutions (Budak-Coşkun, 2009). Since mathematics has a constantly evolving and cumulative structure, the teaching process should also be based on the principle of structuring concepts by relating them to each other (Bingölbali & Özmantar, 2015). Although mathematics is inevitably divided into different subject areas in teaching programs, meaningful connections should be established among these areas, and mathematical knowledge should be presented to students with both intradisciplinary and interdisciplinary coherence and related to life (NCTM, 2000).

Problem solving is considered a core competency in mathematics learning (Schoenfeld, 1985). Rather than involving the mechanical application of procedures, it requires careful examination of the problem, the construction of logical solution strategies, the execution of these strategies, and the critical assessment of the outcomes (Arfé et al., 2020). In this regard, the process can be conceptualized as a sequence of steps that includes understanding the problem, generating and choosing suitable strategies, devising a plan that promotes flexible thinking, implementing the plan, and reflecting on and evaluating the results (Polya, 2014).

This Study

Recent studies have revealed that the interdisciplinary, collaborative, and problem-solving-based structure of the HEM contributes significantly to students' cognitive and affective development (Erdem & Eminoğlu-Küçüktepe, 2024; Orhan, 2025; Tokmak et al., 2023). Numerous studies have demonstrated that HEM has positive effects on students' problem-solving, CT, creativity, collaboration, and 21st-century skills (Çimşir, 2024; Derdiyok et al., 2025). Some studies have also found that the model has positive effects on social-emotional learning, self-awareness, motivation, self-confidence, and school attachment (Ayra et al., 2025; Orhan, 2025). However, findings in this area are mostly based on qualitative data derived from teacher and student opinions. Therefore, more quantitative, longitudinal, and experimental studies on the affective effects of the model are needed. Studies examining teacher opinions (Ayra et al., 2025; Erdem & Eminoğlu-Küçüktepe, 2024; Keskin et al., 2024; Seçer, 2021) generally show that teachers have positive attitudes towards HEM. Teachers have highlighted the model's strengths as its student-centered, collaborative, and interdisciplinary structure. However, they have also frequently emphasized challenges related to infrastructure, materials, time, and planning in practice (Orhan, 2025; Seçer, 2021). Most of the studies reviewed used qualitative designs. While this provides valuable information about the model's effectiveness, it makes it difficult to draw causal inferences.

The majority of studies on HEM in the literature consist of descriptive or opinion-based qualitative studies, and experimental or applied studies focusing on the concrete effects of the model in primary mathematics education are quite limited. While existing studies have shown that the model develops students' social skills such as cooperation, communication, and creative thinking, they do not provide sufficient in-depth analysis of the model's impact on mathematics teaching or the opportunities and challenges teachers encounter in classroom applications. This situation highlights the need for comprehensive, applied research that examines the effects of HEM, particularly in the context of primary school mathematics education.

This study examines how HEM is structured in mathematics education and its multifaceted effects on students' problem-solving skills. It aims to evaluate both teachers' experiences with the application process and the pedagogical contributions of the model. One of the original aspects of the research is that it addresses the integration of HEM into primary-level mathematics lessons from both the teacher

and student perspectives, revealing the impacts of the interdisciplinary method on instructional outcomes. The results are hoped to contribute to teacher instruction but also serve as a guide for practitioners and policymakers to increase the effectiveness of interdisciplinary models. Furthermore, the research provides a comprehensive assessment of the applicability of HEM at the primary education level by revealing primary school teachers' experiences with implementing the model, the difficulties they encountered, and the model's contributions to the teaching process.

The purpose of this study is to examine the integration of HEM into primary school mathematics education and to determine its effect on learners' MPS skills. Accordingly, the study seeks to answer the following research questions.

1. What are the effects of mathematics instruction integrated with HEM-based teaching activities on primary school students' MPS and CT skills?

2. What are the implications of mathematics instruction conducted with HEM-based teaching activities for the MPS teaching process?

METHOD

Research Model

Mixed-methods research refers to an approach in which both quantitative and qualitative data are gathered and analyzed using one or more data collection tools, and the results are subsequently combined and interpreted in an integrated manner (see Figure 1).
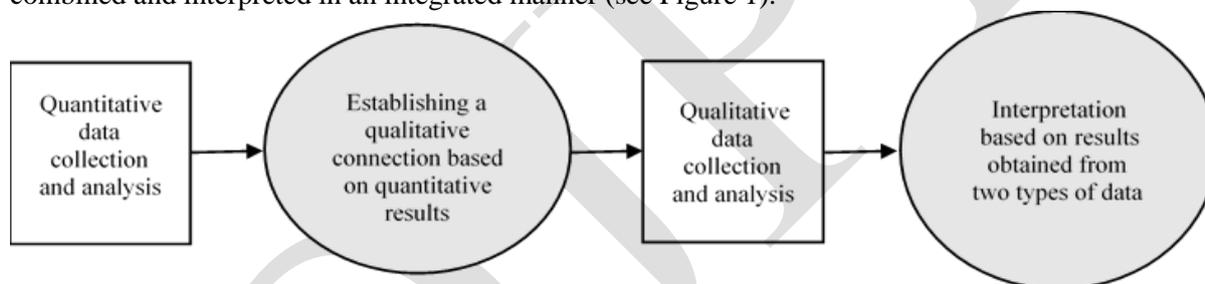
Figure 1. Explanatory sequential mixed-methods approach.

The combined use of quantitative and qualitative methods provides an opportunity to approach and examine the research problem from different perspectives (Niglas, 2010). An explanatory sequential mixed-methods design was utilized in this study. Within this framework, quantitative data were gathered and analyzed in the initial phase, after which qualitative data were collected and examined to further clarify and expand upon the quantitative results. Finally, the outcomes obtained from both phases were combined and interpreted in an integrated manner (Niglas, 2010). In line with this design, the study first focused on the collection and analysis of quantitative data, and subsequently proceeded with the collection and analysis of qualitative data based on the insights gained from the quantitative phase. Consistent with the explanatory sequential design, the qualitative phase was used to gain deeper insight into the factors underlying the quantitative results. For the quantitative component, a quasi-experimental pretest–posttest control group design was employed. The control and experimental groups were formed from intact groups that were comparable with respect to the baseline characteristics identified through pretest measures. While the experimental intervention was applied to the experimental group, the control group continued with the regular instructional process. The impact of the experimental intervention was evaluated by comparing the pretest–posttest score differences between the control and experimental groups (Fraenkel et al., 2012). The qualitative component of the study was structured using a phenomenological research design, which focuses on individuals' lived experiences and perceptions to understand and describe a particular phenomenon or event (Yin, 2016). In this study, the aim was to reveal the reflections of HEM implemented in primary school mathematics lessons on the MPS teaching process through the experiences and perceptions of primary school teachers.

Participants

The study was conducted with 52 fourth-grade students attending a public primary school in Türkiye during the 2024–2025 academic year. For the quantitative component, the sample was selected through convenience sampling, a commonly used approach that enables researchers to access participants efficiently and practically, thereby reducing time and procedural constraints during data collection (Patton, 2002). The extended duration of the HEM-based problem-solving instruction, the adequate facilities of the school where the implementation was conducted, and the preparatory requirements of the activities are among the reasons for choosing this sampling method.

One of the four branches available at the school was randomly selected as the experimental group, and another was designated as the control group. There were 26 students in the experimental group, where HEM-based instruction activities were implemented, and 26 learners in the control group, which continued with curriculum-based traditional instruction. Thus, the research group consisted of a total of 52 learners. The ages of the students in the control and experimental groups ranged from 11 to 12 years. Furthermore, according to information obtained from primary school teachers, the students in both groups shared similar characteristics in terms of socioeconomic status (income level, parents' educational level, living conditions, access to social services, and access to technological devices), as well as academic achievement. Particular attention was paid to ensuring that the experimental group (54% female; 46% male, 26 students) and the control group (42% female; 58% male, 26 students) were balanced in terms of number, gender, and the variables examined.

During the qualitative phase of the research, semi-structured interviews were conducted with 15 primary school teachers who implemented HEM-based instructional activities in MPS teaching. The teachers' perceptions of the HEM approach and their teaching experiences were explored through these interviews. To identify the teachers included in the interviews, criterion sampling -a purposive sampling strategy- was employed (Creswell & Clark, 2017). The HEM approach in the mathematics teaching process and participation in practical training related to this model were used as criteria, and the opinions of 15 primary school teachers were obtained.

All necessary official permissions were obtained prior to including students in the study, and no financial incentives or payments were provided. Participation in the study was entirely voluntary. The study adhered to the ethical principles of the Declaration of Helsinki and the guidelines established by the Turkish Council of Higher Education. Approval for both the implementation and data collection processes was obtained from the Ethics Committee of Bayburt University on May 07, 2025 (Decision No: 195; Session No: 5). Prior to data collection, participants and their legal guardians were informed about the aim of the study, the voluntary nature of participation, confidentiality procedures, and their right to withdraw at any time without any negative consequences. Following this process, written informed consent was secured from both participants and their guardians. All data were collected and used solely for scientific purposes and were analyzed anonymously.

HEM-Based Teaching Activities

In this study, the teaching process was conducted through HEM-based activities. First, based on the mathematics course curriculum, two subjects that could be associated with HEM were identified for the grade level. Two lesson plans were prepared based on these subjects (see Table 2) and the process follow-up guide (see Fig. 2). Similar studies using the HEM approach (Çimşir, 2024; Derdiyok et al., 2025; Tokmak, 2023) were examined and used as a guide in preparing the lesson plans. A process-tracking guide was prepared to implement the lesson plans systematically (see Fig. 2). The guide was shaped by considering the problem-solving stages and the Harezmi Application Guide prepared by the Ministry of National Education. In this study, Polya's problem-solving stages (understanding the problem, planning the solution, implementing the plan, and checking the solution) were considered (Polya, 2014).
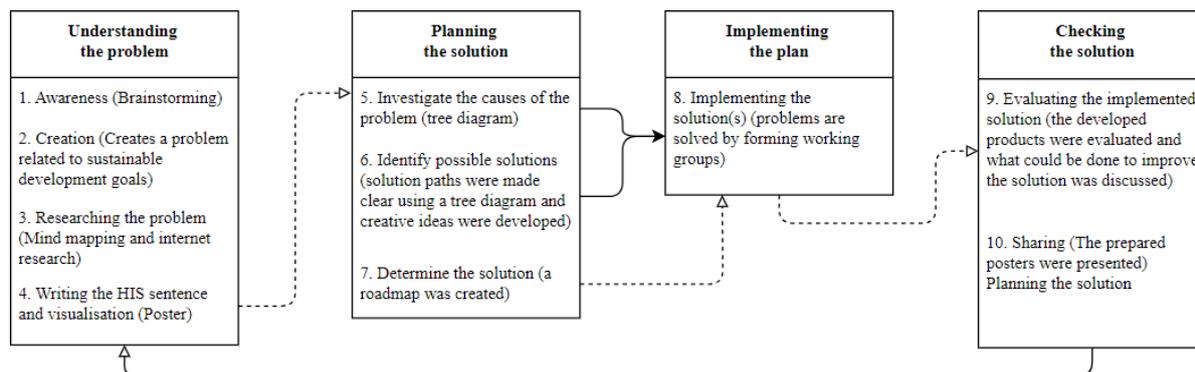
Figure 2. Process tracking guide used in the problem-solving teaching process.

Implementation Process

Before the instructional process began, a pretest was administered to examine whether the control and experimental groups differed significantly in terms of problem-solving and CT skills. The pretest data were examined using SPSS 25.0, and the results indicated that there was no statistically significant difference between the groups. To evaluate differences in group performance, an independent-samples t-test was applied. The intervention was implemented over a three-week period, comprising 12 class hours, in line with the instructional plan outlined in Table 2. Each instructional session lasted 40 minutes. The intervention procedures were carried out concurrently in both the control and experimental groups.

Two lesson plans were prepared at the fourth-grade level, taking into account the learning outcomes related to "addition" and "symmetry." Within this scope, lesson plans were prepared on solving problems requiring addition and drawing the symmetry of a given shape. When determining the learning outcomes, attention was paid to their alignment with the items in the HEM process follow-up guide. The prepared lesson plans were implemented identically in both groups. All lessons were conducted according to course plans designed in advance by the investigator and structured based on the learning outcomes. In the control group, the teaching process was conducted solely based on the textbook. The teaching activities for the control and experimental groups are presented in Table 1.

Table 1. Teaching activities implemented in the control and experimental groups.

	Experimental group	Control group
Pre-test	PSS, CTTS	
Learning process (three week)	**Addition**	
	The students brainstormed ideas on environmental issues and examined a problem situation related to the Sustainable Development Goals through group work, focusing on the damage caused to the environment by litter in the school garden. Mind maps and tree diagrams were used during the process to identify the causes of the problem, and possible solutions were discussed to develop creative ideas.	Lessons were teacher-centred; topics were explained using the textbook's explanations, sample solutions and demonstrations on the board. The addition process was reinforced through standard exercises and routine problem examples.
	Symmetry	
	Students worked in heterogeneous groups, conducting guided research, mind mapping and visualisation activities to express their love of symmetry and nature through posters, banners and slogans. Subsequently, analysis, measurement and application studies were carried out on objects found in nature; symmetry lines were drawn and geometric shapes were created.	The topic of symmetry was addressed by drawing symmetry lines on geometric shapes and examining ready-made shapes. Students worked individually, and the learning process was completed mainly through question-and-answer sessions, explanations, and workbook activities.
Post-test	PSS, CTTS, SSIF	

HEM-based instruction activities were designed using an interdisciplinary approach, with consideration of the process guide prepared by the researchers. The developed activities were reviewed by two mathematics education experts and evaluated for their alignment with HEM, the instructional content, and the characteristics of the target student group. In line with the experts' feedback, the 5E instructional model was incorporated into the design and implementation of the lesson plans. Subsequently, a pilot application of a lesson plan was carried out with a different class with similar characteristics that was not included in the study. After the pilot application, it was decided to conduct instruction in small groups rather than as a whole-class activity. Information about HEM-based teaching activities is presented below.

The application was carried out through small-group teaching led by the first researcher, with the second researcher, a primary school teacher, and two graduate students. The graduate students studying in the field of primary school education are also experienced primary school teachers. First, the learners in the experimental group were divided into four groups. The first researcher was responsible for teaching, while the other teachers served as group leaders. The implementation took place during the spring term of the academic year. Instruction was delivered through HEM-based activities in the experimental group, while the control group received traditional instruction using the mathematics teaching program. To prevent implementer bias, the measurement tools used to evaluate the process and outcome were scored objectively in accordance with standard criteria. In addition, the researcher benefited from the opinions and contributions of field experts in the analysis of qualitative data.

Table 2. HEM-based teaching activities.

Subject	Content	HEM duty
Problem solving requiring addition	Brainstorming is conducted on the question, "What are environmental problems?" A problem situation related to Sustainable Development Goals (SDG) (garbage in the schoolyard harming the environment) is presented. Students conduct research in groups, create mind maps, and express their ideas through HIS (a real-world problem) sentences, posters, or slogans.	1. Notice 2. Create 3. Research 4. Visualize
	Students first use a tree diagram to identify the root causes of the problem. They then discuss possible solutions using the diagram to develop creative ideas.	5. Identify the causes 6. Develop solutions
	Students walk around the garden on the school map, collect trash, mark the trash they find on the map, and number it. They create and solve their own math problems by measuring the distances between pieces of trash. Geometric shapes are made from the collected trash, and slogans are created.	7. Apply
	Students present the products they have developed (posters, slogans, geometric shapes) in class.	8. Check
Symmetry	Students are asked about symmetrical structures in nature and are told a story about symmetry. A problem situation is presented using the example of Elif and Mert's birdhouse design. Students conduct research in heterogeneous groups with guidance, visualize their love of nature and symmetry using mind maps, and express their ideas through banners, posters, or slogans.	1. Notice 2. Create 3. Research 4. Visualize
	Students analyze symmetrical shapes and patterns found in nature. Possible solutions and applications are determined using lines of symmetry and geometric shapes.	5. Identify the causes 6. Develop solutions
	Students go outdoors and draw symmetry lines on rocks, trees, or other objects to create geometric shapes. In the symmetry map activity, groups apply the shapes to each other using strings or drawings. In the birdhouse problem, they determine the position of the window based on the symmetry line and measurements; they draw and implement their designs.	7. Apply
	Students present their symmetry stones, drawings, and models found in nature in the exhibition area.	8. Check

Lesson plans were prepared and implemented based on the 5E teaching model. At the beginning of the lesson, students engaged in brainstorming with questions about environmental problems and symmetrical structures to capture their attention, and they became interested in the problem situation through the symmetry story and the birdhouse example (Engage). Then, in heterogeneous groups, with their instructors' guidance, they conducted research and visualized environmental pollution, love of nature, and symmetry using the mind map method (Explore).

Figure 3. Images from HEM-based teaching activities --Dünya temiz bir dairedir [The world is a clean apartment]; Çevre, Toprak, Su, Hava, ve Gürültü Kirliliği [Environmental, Soil, Water, Air, and Noise Pollution]--

Based on the information they obtained, the groups discussed the problem's causes and possible solutions, and created a plan using tree diagrams or symmetry lines (Explain). They then collected trash in the schoolyard or nature through hands-on activities, produce geometric shapes and symmetry models, and implemented the plan through symmetry map activities and mathematical measurements

(Elaborate). Finally, students presented the posters, slogans, and symmetry models they had prepared in the exhibition area, evaluated their products and solutions, and reinforced their learning by discussing shortcomings (Evaluate). Images from the implementation process are presented Figure 3.

Data Collection Instruments

Quantitative and qualitative data collection instruments were employed in a complementary way to enhance data diversity. Accordingly, a problem-solving and CT scale was utilized as the quantitative instrument, while a semi-structured interview form served as the qualitative data collection tool.

Problem solving scale (PSS)

In this study, the PSS for primary schools, developed by Değirmenci and Deringöl (2024), was used to evaluate primary school learners' problem-solving abilities. The scale, prepared to assess students' problem-solving skills, consists of a total of 16 items. The scale, consisting of three factors, covers the dimensions of "self-assessment in problem solving," "problem solving knowledge," and "metacognition in problem solving." The scale items, which have a five-point Likert-type rating, are answered with the options "Always," "Often," "Sometimes," "Rarely," and "Never." Students responded to the scale items by selecting the option that best suited them. The scale scores range from 16 to 80, with lower scores indicating weaker problem-solving skills and higher scores indicating stronger problem-solving skills. The item factor loadings of the scale range from .39 to .75. As a result of the confirmatory factor analysis (CFA) conducted to assess construct validity, it was determined that fit indices, such as χ^2/df, RMR, RMSEA, SRMR, CFI, IFI, GFI, and AGFI, were at acceptable levels. Furthermore, the Cronbach's Alpha coefficients for the scale's subscales were .85 for self-assessment in problem solving, 0.84 for problem-solving knowledge, and .75 for metacognition in problem solving. These values indicate that the scale has high reliability and is a valid measurement tool.

Critical thinking tendencies scale (CTTS)

The CTTS (Uluçinar & Akar, 2021) was used to determine students' CT skills. This scale, consisting of four factors and a total of 18 items, was prepared for primary school students and is organized in a four-point Likert scale format. The items are organized in a four-point Likert scale format with the options "Never," "Sometimes," "Most of the time," and "Always." The item factor loadings range from .35 to .74. The scores on the scale range from 18 to 72. The scale was administered to primary school students under the guidance of an instructor. A CFA was conducted to examine the goodness-of-fit of the measurement model. The findings indicated excellent fit for the χ^2/df, RMR, RMSEA, SRMR, and IFI indices, while the CFI and TLI values demonstrated an acceptable level of model fit. Furthermore, the overall reliability of the scale was assessed using Cronbach's alpha, which the researchers calculated at .74.

Semi-structured interview form (SSIF)

In the present study, SSIFs were conducted with 15 primary school teachers who implemented the HEM approach in their mathematics lessons. The interview format permits the inclusion of probing or follow-up questions alongside predetermined questions, allowing greater flexibility during data collection (Fraenkel et al., 2012). The interview questions were reviewed by two mathematics education faculty members, who confirmed that the interview form was appropriate for the participants' level, the model, and the implementation process. Furthermore, the interview form was reviewed and deemed appropriate by the primary school teachers at the school where the application was implemented. In addition, the prepared form was piloted with primary school teachers from a different school not included in the application. During the pilot study, questions that teachers found difficult to answer were revised accordingly with the support of the experts consulted during the form's preparation and supplemented with alternative questions. The interview form consists of eight open-ended questions. The interviews were conducted face-to-face with the teachers by the second researcher and lasted approximately 20 minutes each. The interviews were recorded with a voice recorder, and the recordings were transcribed into written text and prepared for analysis.

Data Analysis

Quantitative analyses were conducted using SPSS version 25. Given that multiple dependent variables were examined simultaneously, MANOVA was employed (Smith et al., 2020). Before conducting MANOVA, key assumptions—including adequate sample size, normality, the presence of outliers, linearity and multicollinearity, singularity, and the homogeneity of variance–covariance matrices—were assessed in accordance with the guidelines outlined by Pallant (2020). The analyses revealed that the data met the assumptions and that the sample sizes were sufficient for each dependent variable. Normality was assessed using histograms, skewness and kurtosis values, and the Levene test ($p > 0.05$). Multivariate normality was assessed using Mahalanobis distance values. These values were compared against the chi-square distribution, and the results indicated that none of the observations exceeded the critical threshold (χ^2 (2, $p < .001$) = 13.82). This result indicates that the data met the assumption of multivariate normality. In addition, correlation coefficients indicated that the intercorrelations among variables remained below the .80 threshold. Visual inspection of distribution plots further supported the assumption of linearity. The results of Box's M test ($p > .05$) confirmed the homogeneity of variance–covariance matrices, indicating that the data met the assumptions required for MANOVA (Smith et al., 2020).

Qualitative data were gathered via face-to-face teacher interviews and analyzed through content analysis. Researchers coded the data and created categories and subcategories based on common characteristics. Another researcher then checked the resulting code matrix, and inter-coder reliability was calculated as .83 (Miles & Huberman, 1994). The in vivo coding method, based on direct quotations from participant statements, was preferred for reporting the obtained codes and categories (Saldana, 2021). At the outset of the study, analyses indicated that the control and experimental groups were equivalent in terms of pretest scores, with no statistically significant differences observed across any of the variables ($p > .05$). Furthermore, to protect participant confidentiality, teachers were assigned code names, and findings were presented using these code names. Fifteen primary school teachers were coded as HEM-T1, HEM-T2, HEM-T3…, HEM-T15 according to their sequence numbers.

RESULTS

A MANOVA was applied to examine the effect of HEM-based teaching activities integrated into primary school mathematics lessons on students' MPS and CT tendencies. Certain preconditions of MANOVA (linearity, normality, homogeneity of variance-covariance matrices, and multicollinearity) must be met. In this context, Box's M test was not significant ($p > .05$). This indicates that the condition of homogeneity of variance-covariance matrices was met.

According to the analysis of test results, descriptive statistics reveal that the experimental group scored higher in MPS (M = 69.23, SD = 6.19) and CT (M = 59.53, SD = 4.72). The descriptive statistics data for the control group were lower than those for the experimental group in MPS (M = 59.15, SD = 6.05) and CT (M = 44.57, SD = 7.04). To examine whether group differences were statistically significant, multivariate comparisons were conducted using MANOVA, and the results are presented below (see Table 3).

Table 3. MANOVA results related to CT and MPS.

Tests	Multiple test	Value	Mean square	*df*	*F*	*p*	Partial eta squared
Post-test	Wilks' λ	.361	-	-	43.457	.000	.639
	Dependent Variables						
	CT	-	2910.019	1	80.887	.000	.618
	MPS	-	1320.077	1	35.146	.000	.413

Analyses based on posttest scores revealed a statistically significant difference between the control and experimental groups, favoring the experimental group ($F_{(1, 50)}$ = 43.457, p = .000; Wilks' λ = .361;

η^2 = .639). In other words, the HEM-based teaching activities had been implemented had a significant effect on students' CT and MPS skills. Moreover, the experimental group demonstrated significantly higher CT levels than the control group ($F_{(1, 50)}$ = 80.887, p = .000, η^2 = .618). Likewise, the experimental group achieved significantly higher MPS scores than the control group ($F_{(1, 50)}$ = 35.146, p = .000, η^2 = .413). According to Cohen's (2013) classification, the calculated eta-squared (η^2) values were determined to have large effect sizes.

Results Regarding Sub-Factors of CT Tendencies

A MANOVA was performed to compare the posttest scores of the control and experimental groups across the subdimensions of CT tendencies, and the results are presented below (see Table 4).

Table 4. MANOVA results for sub-factors of CT tendencies.

Tests	Multiple test	Value	Mean square	df	F	p	Partial eta squared
	Wilks' λ	.245	-	-	36.203	.000	.755
	Dependent Variables						
Post-test	Skepticism	-	204.019	1	107.947	.000	.683
	Curiosity	-	138.942	1	19.420	.000	.280
	Open mindedness	-	129.308	1	23.308	.000	.318
	Biasness	-	272.327	1	94.331	.000	.654

The analysis results presented in Table 4 revealed statistically significant differences between the control and experimental groups on the sub-factors of the CTTS ($F_{(1, 50)}$ = 36.203, Wilks' λ = .245, p = .000, η^2 = .755). Furthermore, the results for the doubt sub-dimension showed a significant difference in favor of the experimental group ($F_{(1, 50)}$ = 107.947, p = .000, η^2 = .683). When examining the descriptive statistics for the doubt subscale, the experimental group (M = 13.88, SD = 1.27) had significantly higher scores than the control group (M = 9.92, SD = 1.46). Similarly, the results for the curiosity subscale showed a significant difference in favor of the experimental group ($F_{(1, 50)}$ = 19.420, p = .000, η^2 = .280). When examining the descriptive statistics for the curiosity subscale, the experimental group (M = 16.42, SD = 2.26) had higher scores than the control group (M = 13.15, SD = 3.02). On the other hand, the results for the open-mindedness subscale showed a significant difference in favor of the experimental group ($F_{(1, 50)}$ = 23.308, p = .000, η^2 = .318). When examining the descriptive statistics for the open-mindedness subscale, the experimental group (M = 16.15, SD = 2.18) had higher scores than the control group (M = 13.00, SD = 2.51). Finally, the results for the objectivity (Biasness) subscale showed a significant difference in favor of the experimental group ($F_{(1, 50)}$ = 94.331, p = .000, η^2 = .654). When examining the descriptive statistics for the objectivity subscale, the experimental group (M = 13.07, SD = 1.54) had higher scores than the control group (M = 8.50, SD = 1.83).

Results Related to MPS Sub-Factors

MANOVA was conducted to examine differences between the control and experimental groups in posttest scores across the subdimensions of MPS skills, and the results are reported below (see Table 5).

Table 5. MANOVA results for subfactors of MPS.

Tests	Multiple test	Value	Mean square	df	F	p	Partial eta squared
	Wilks' λ	.542	-	-	13.503	.000	.458
	Dependent Variables						
Post-test	Self-assessment in problem-solving	-	330.019	1	39.780	.000	.443
	Problem-solving knowledge	-	76.327	1	14.287	.000	.222
	Metacognition in problem solving	-	88.923	1	16.100	.000	.244

The results displayed in Table 5 indicate statistically significant differences between the control and experimental groups across the subdimensions of the problem-solving scale ($F_{(1, 50)}$ = 13.503, Wilks' λ = .542, p = .000, η^2 = .458). Furthermore, the results for the self-assessment sub-dimension of problem solving showed a significant difference in favor of the experimental group ($F_{(1, 50)}$ = 39.780, p = .000, η^2 = .443). When examining the descriptive statistics for the self-assessment sub-dimension in problem solving, it was observed that the experimental group (M = 36.57, SD = 2.92) had significantly higher scores than the control group (M = 31.53, SD = 2.83). Similarly, the results for the problem-solving knowledge sub-dimension showed a significant difference in favor of the experimental group ($F_{(1, 50)}$ = 14.287, p = .000, η^2 = .222). When examining the descriptive statistics for the problem-solving knowledge sub-dimension, it was determined that the experimental group (M = 16.03, SD = 2.45) had higher values than the control group (M = 13.61, SD = 2.15). Finally, the results for the metacognition sub-dimension in problem solving showed a significant difference in favor of the experimental group ($F_{(1, 50)}$ = 16.100, p = .000, η^2 = .244). When examining the descriptive statistics for the metacognition sub-dimension of problem solving, the experimental group (M = 16.61, SD = 2.31) had higher scores than the control group (M = 14.00, SD = 2.38).

Results from Teacher Interviews

Findings from teacher interviews are presented under two themes: "Findings related to the HEM-supported mathematics teaching process" and "Findings related to the MPS process."

Results Related to the HEM-Supported Mathematics Teaching Process

Findings related to the HEM-supported mathematics teaching process are presented in Table 6 under three categories: "opportunities," "challenges," and "recommendations."

Table 6. Findings related to the HEM-supported mathematics teaching process.

Category	Code	Teacher	f	%
Opportunities	Develops problem-solving skills	HEM-T1, HEM-T2, HEM-T3, HEM-T4, HEM-T5, HEM-T6, HEM-T9, HEM-T10, HEM-T11, HEM-T12, HEM-T15	11	20
	Provides an interdisciplinary learning environment	HEM-T1, HEM-T2, HEM-T4, HEM-T5, HEM-T6, HEM-T8, HEM-T10, HEM-T13, HEM-T11, HEM-T14	10	18
	Enables connection to daily life	HEM-T1, HEM-T2, HEM-T4, HEM-T6, HEM-T7, HEM-T8, HEM-T10, HEM-T11, HEM-T13, HEM-T14	10	18
	Encourages active participation	HEM-T1, HEM-T2, HEM-T4, HEM-T5, HEM-T6, HEM-T10, HEM-T11, HEM-T13	8	14
	Enables meaningful and lasting learning	HEM-T1, HEM-T2, HEM-T3, HEM-T4, HEM-T5, HEM-T7, HEM-T11	7	12
	Supports social learning	HEM-T2, HEM-T3, HEM-T4, HEM-T7, HEM-T11, HEM-T14	6	11
	Enables technology-supported learning	HEM-T1, HEM-T2, HEM-T5, HEM-T6	4	7
Challenges	Lack of resources and materials	HEM-T2, HEM-T3, HEM-T4, HEM-T5, HEM-T6, HEM-T10, HEM-T12, HEM-T13	8	27
	Limited teacher competencies related to the model	HEM-T1, HEM-T3, HEM-T4, HEM-T5, HEM-T6, HEM-T11, HEM-T13	7	23
	Time limit	HEM-T1, HEM-T3, HEM-T5, HEM-T8, HEM-T10, HEM-T11	6	20
	Difficulty in planning lessons	HEM-T3, HEM-T6, HEM-T10, HEM-T12	4	13
	Uncertainties in the measurement and evaluation process	HEM-T1, HEM-T2, HEM-T4	3	10
	Lack of technological infrastructure	HEM-T1, HEM-T3	2	7

Table 6 (Continued). Findings related to the HEM-supported mathematics teaching process.

Category	Code	Teacher	f	%
	Delivery of teacher training	HEM-T1, HEM-T2, HEM-T5, HEM-T9, HEM-T10, HEM-T11, HEM-T12, HEM-T14	8	20
	Adaptation to the curriculum	HEM-T3, HEM-T5, HEM-T6, HEM-T7, HEM-T13, HEM-T14, HEM-T15	7	18
	Provision of resources and materials	HEM-T1, HEM-T2, HEM-T3, HEM-T10, HEM-T11, HEM-T12	6	15
	Sharing successful examples	HEM-T2, HEM-T3, HEM-T7, HEM-T11, HEM-T12	5	13
Recommendations	Increasing collaborations (universities, NGOs, private organizations)	HEM-T1, HEM-T2, HEM-T10, HEM-T11	4	10
	Implementing a flexible program	HEM-T2, HEM-T4, HEM-T9	3	8
	Strengthening technological infrastructure	HEM-T2, HEM-T12, HEM-T14	3	8
	Increasing support from school administrations	HEM-T1, HEM-T3, HEM-T8	3	8

Table 6 shows that HEM-supported mathematics instruction offers students many opportunities. The vast majority of teachers stated that the model improved students' problem-solving skills (f=11). This may be because HEM-supported mathematics instruction provides students with learning environments based on real-world, interdisciplinary problems. Furthermore, teachers indicated that HEM-supported mathematics teaching provides an interdisciplinary learning environment (f=10) and helps relate what is learned to daily life (f=10). In addition, the model was observed to ensure active student participation (f=7). Collaborative learning environments and group work within the model may have contributed to students' more effective participation in the process. According to teachers' views, HEM makes significant contributions to developing students' participation and problem-solving skills. HEM-T11 commented on this situation: *"... I have just started applying the model and noticed that its biggest advantage is increasing students' active participation."* Similarly, HEM-T10 emphasized that the model supports students' cognitive and social skills, stating: "*The model offers great opportunities to equip students with analytical thinking, collaboration, and MPS skills in primary mathematics education.*" Furthermore, HEM-T12 stated, "*... thanks to the model, students can apply problem-solving steps and develop problem-solving habits,*" indicating that the model contributes to students' systematic problem-solving processes. HEM-T11 stated, "*Students who produce solutions by working with different disciplines become more creative not only in mathematics but also in solving problems in daily life.*"

In the difficulties category, a significant proportion of teachers reported a lack of resources and materials (f=8) during the HEM-supported mathematics teaching process. This situation can be attributed to inadequate technological infrastructure and the limited availability of materials for lessons. Another difficulty expressed by teachers was limited teacher competence regarding the model (f=7). A large proportion of teachers reported difficulties in implementation because they could not fully grasp the model's philosophy and application processes. Additionally, teachers stated that they experienced difficulties with time management (f=7). The participants' views exemplify how the difficulties encountered in implementing the model are reflected in the process. HEM-T10 drew particular attention to time management, stating, *"... but implementing the model also involves some difficulties. One of the biggest difficulties is time management. Compared to traditional teaching, process-oriented learning requires more time.*" HEM-T12 emphasized the lack of materials, stating, *"... a lack of appropriate teaching methods and tools may be among the difficulties encountered.*" Additionally, HEM-T13 emphasized that teachers experienced difficulties in implementing the model due to its novelty, stating, "*Teachers are experiencing difficulties in implementation because it is a new approach.*"

In the recommendations category, teachers offered various suggestions for more effectively implementing the model in the mathematics teaching process. The vast majority of teachers emphasized the need for teacher training (f=8) to ensure the sustainable implementation of HEM. Such training can facilitate implementation by increasing teachers' proficiency with the model. It was also noted that it is important to make the model more compatible with the current curriculum (f=7). This can prevent difficulties teachers may encounter in lesson planning. In addition, teachers stated that resource and material support should be increased (f=6) for a more efficient application in the HEM-supported mathematics teaching process. Another suggestion to support the dissemination of the model is to share successful application examples (f=5). Teachers stated that they could carry out the process more effectively by benefiting from the experiences of colleagues who have previously implemented the model successfully. These suggestions show that teachers have concrete expectations regarding increasing the applicability of HEM. An examination of the participants' suggestions reveals various views on the dissemination and development of the model. HEM-T13 emphasized that the model should be integrated into all schools, stating, "*It should be implemented not only in pilot schools but in all schools.*" HEM-T14 drew attention to the importance of curriculum-aligned integration, "*Work can be done on integrating the curriculum.*" Furthermore, HEM-T11 stated, *"...a system could be created where experienced teachers could mentor new teachers. This would help the model to be implemented correctly and effectively in the classroom,*" indicating that establishing a mentoring system among teachers would facilitate implementation.

Results Related to the MPS Process

Findings related to the MPS process are presented in Table 7 under two categories: "cognitive skills" and "social and affective skills."

Table 7. Findings related to the MPS process.

Category	Code	Teacher	f	%
Cognitive skills	Develops different strategies for solutions	HEM-T1, HEM-T2, HEM-T3, HEM-T4, HEM-T5, HEM-T6, HEM-T7, HEM-T8, HEM-T9, HEM-T11, HEM-T12, HEM-T13, HEM-T14	13	35
	Supports analytical thinking	HEM-T1, HEM-T2, HEM-T5, HEM-T6, HEM-T12, HEM-T14	6	16
	Encourages creative thinking	HEM-T1, HEM-T3, HEM-T4, HEM-T6, HEM-T10, HEM-T12	6	16
	Establishes cause-and-effect relationships	HEM-T1, HEM-T2, HEM-T6, HEM-T8, HEM-T11	5	14
	Develops algorithmic thinking	HEM-T1, HEM-T4, HEM-T10, HEM-T11, HEM-T14	5	14
	Provides a critical perspective	HEM-T9, HEM-T10	2	5
Social and emotional skills	Supports collaborative problem-solving	HEM-T1, HEM-T3, HEM-T4, HEM-T10, HEM-T11, HEM-T13, HEM-T15	9	26
	Increases interaction with the class and teacher	HEM-T1, HEM-T2, HEM-T3, HEM-T4, HEM-T5, HEM-T12, HEM-T13	9	26
	Increases interest and motivation in math class	HEM-T1, HEM-T2, HEM-T3, HEM-T4, HEM-T5, HEM-T10, HEM-T11, HEM-T13	8	24
	Facilitates communication and collaboration among students	HEM-T2, HEM-T4, HEM-T5, HEM-T10, HEM-T15	5	15
	Develops a positive attitude toward mathematics	HEM-T1, HEM-T3, HEM-T7	3	9

When Table 7 is examined, under the cognitive skills category, the majority of teachers stated that HEM-supported mathematics instruction contributed to students develop different strategies for solving problems during the problem-solving process (f=13). Furthermore, in this category, teachers indicated that the model supported students' analytical and creative thinking (f=6). The interdisciplinary and project-based structure of the model may contribute to diversifying students' thinking processes and developing a systematic approach. HEM-T10 stated, "The model supports the development of not only mathematical competencies but also essential skills such as critical thinking,

creativity, communication, and cooperation." This shows that the model supports students' multidimensional development and contributes not only to academic but also to social skill acquisition. HEM-T11 emphasized that the model increases innovative thinking and productivity skills in students, stating, "*The model encourages students to think creatively by offering them different perspectives. Especially in activities such as coding and robotics, students work to develop more innovative and original solutions.*" HEM-T13 explains the model's purpose and scope, stating: "*The HEM encourages students to solve problems collaboratively, helping them develop algorithmic thinking as well as cognitive and social skills, while also strengthening their critical thinking.*"

In the social and emotional skills category, teachers indicated that HEM-supported mathematics instruction increased classroom interaction (f=9), contributed to collaborative problem-solving (f=9), and increased students' interest and motivation in the lesson (f=8). This finding indicates that HEM-supported teaching not only promotes cognitive development but also provides a structure that supports social learning. Participant views support these findings. For example, HEM-T10 stated, "*Instead of solving problems individually, students develop different perspectives by discussing them in groups,*" indicating that students develop different ways of thinking through group discussions. Similarly, HEM-T13 emphasized that the model offers a teamwork-based learning environment, stating, "*Ideas are generated as a team to solve problems.*" Furthermore, HEM-T15: "*Students learn to relate topics to different disciplines and achieve common goals by working together,*" highlighting that interdisciplinary interaction develops students' ability to achieve common goals.

DISCUSSION, CONCLUSION, and RECOMMENDATIONS

This study investigated the impact of HEM-based instructional activities incorporated into primary school mathematics lessons on students' MPS and CT dispositions. Accordingly, the experimental group received instruction through HEM-based activities, whereas the control group followed the standard curriculum. To assess the effectiveness of the instruction, pretest and posttest measurements were administered to both groups, and the pretest results indicated that the groups were comparable across all variables at the outset.

In this study, the test averages indicated that HEM-based teaching activities had a significant impact on the MPS and CT skills of primary school students. The fact that the experimental group's scores for both MPS and CT were significantly higher than those of the control group indicates that the activities implemented were effective in developing students' thinking processes. This can be explained by the HEM approach, which enables students to develop different perspectives and actively participate in the processes of interpreting and restructuring knowledge. In particular, the large effect sizes observed in experimental studies indicate that HEM activities led to substantial improvements in students' cognitive skills. Similar results have been obtained in other studies in the literature examining the effect of HEM-supported mathematics instruction on MPS and CT skills. This finding is consistent with the results of the study conducted by Tokmak et al. (2023). In that study, a significant improvement in perceptions of problem-solving skills was also observed among students who participated in the HEM process. Similarly, the themes of "personal development," "social learning," and "educational gains" that emerged in the study by Derdiyok et al. (2025) indicate that HEM applications support students' problem-solving, mental development, and idea-generation skills. Furthermore, studies conducted by Orhan (2025) and Kök (2024) also revealed that HEM positively contributes to problem-solving skills by increasing student participation. Seçer (2021) reported that teachers had positive views of the HEM application, supporting the model's applicability in teaching processes. No studies were found in the literature regarding the model's support for CT skills. In this context, when examining STEM studies similar to the model, such interdisciplinary approaches have been shown to develop the CT skills of primary school students (Chen et al., 2023; English, 2023; Küçük et al., 2023; Tang et al., 2025; Weng et al., 2022).

Mathematical Problem Solving

In the study, the subdimensions of the problem-solving scale were examined and the experimental group students scored significantly higher than the control group on the self-assessment, problem-solving knowledge, and metacognition. This finding shows that HEM-based teaching activities not only increase overall problem-solving success but also improve students' performance in different dimensions of problem-solving. This result reveals that HEM enables students to become aware of their own problem-solving processes, review their strategies, and strengthen their solution evaluation skills. Research in the literature also supports these findings. These findings largely coincide with other studies examining the effect of HEM-based applications on problem-solving skills (Ayra et al., 2025; Kök, 2024). Studies have shown that HEM applications significantly improve students' problem-solving skills (Erdem & Eminoğlu-Küçüktepe, 2024). Similarly, in the study by Çimşir et al. (2022), students stated that HEM activities improved their problem-solving and collaborative work skills; they indicated that these activities made a significant contribution to stages such as "identifying problems from real life, collecting data, forming hypotheses, and testing them." The study by Çimşir (2024) also found that the experimental group showed significant development in problem-solving skills, confidence in problem-solving skills, and avoidance subdimensions. However, the lack of a significant difference in the self-regulation subdimension in the same study suggests that the model had a more limited effect on some cognitive components. Furthermore, the study by Tokmak et al. (2023) found that HEM directly increased students' metacognitive awareness levels, which directly supports the metacognitive development findings obtained in this study. These similarities and partial differences between the studies can be explained by differences in scope, sample level, and application duration. For example, in Çimşir's (2024) study, the HEM application was conducted over a short period of eight weeks, which may have prevented the observation of meaningful changes in skills requiring longer-term experience, such as self-regulation and perception of success. In contrast, the application process in this study, structured around students' active participation, may have led to meaningful developments across different sub-dimensions of problem-solving. Furthermore, the interdisciplinary and interaction-based structure of HEM may have supported metacognitive awareness by allowing students to review and organize their own thinking patterns during the problem-solving process.

Critical Thinking

When examining the sub-dimensions of the CTTS in the study, it was observed that the experimental group students scored significantly higher than the control group in the dimensions of skepticism, curiosity, open-mindedness, and objectivity. This finding shows that HEM-based teaching activities develop students' CT tendencies not only in general terms but also in different dimensions. No study directly examining the effect of HEM on CT skills in mathematics education was found in the literature. However, results from studies using holistic, interdisciplinary, and problem-based approaches, such as STEM and STEAM, are consistent with these findings (English, 2023; Küçük et al., 2023; Weng et al., 2022). Research findings indicating that problem-based and project-based learning processes increase students' curiosity, questioning, and open-mindedness (English, 2023) are consistent with those of this study. This similarity can be explained by HEM's approach, which centers the student in the learning process and structures CT as a natural learning process. Studies using similar approaches in the literature also show that the development of CT tendencies is associated with experiences such as active student participation, group-based problem-solving activities, and the production of original projects (Charlton & Avramides, 2016; Hu et al., 2020). In this study, interdisciplinary interaction, digital tool use, and student collaboration, which are among the basic principles of HEM, may have supported the development of the skepticism, curiosity, and objectivity dimensions of CT (Chen et al., 2023; Tang et al., 2025). Furthermore, the process-oriented structure of HEM enables students not only to access information but also to develop their ability to analyze information and evaluate different perspectives.

Teacher Opinions

According to teacher opinions, this study found that HEM-supported teaching in the opportunities category improved students' problem-solving skills, provided interdisciplinary learning environments, and facilitated the connection of learned knowledge to daily life. These findings are consistent with the studies conducted by Koçoğlu (2018) and Ceylan et al. (2020). Both studies indicate that HEM enables students to learn by doing and experiencing, participate in interdisciplinary work, and use the information they learn in daily life. Similarly, other studies have also reported that HEM applications strengthen students' problem-solving skills and positively affect their approach to problems in daily life (Ayra et al., 2025; Babayiğit-Durakcan, 2024; Erdem & Eminoğlu-Küçüktepe, 2024; Keskin et al., 2024; Kök, 2024). These results are consistent with the theoretical framework of HEM. The three fundamental pillars of HEM—interdisciplinary approach, collaborative learning, and problem-solving-based learning strategies—aim to develop students' self-awareness, communication, and problem-solving skills (Çeliköz & Hastürk, 2024). Furthermore, integrating educational technologies reinforces these gains and enhances students' ability to identify and solve problems in their daily lives, thereby supporting the model's effectiveness in both theoretical and practical terms (Ceylan et al., 2020).

In the difficulties category, material and resource shortages in HEM-based teaching applications, limited teacher competencies, and time management difficulties were observed. Similar findings have been obtained in studies on the difficulties encountered in HEM applications (Ayra et al., 2025; Erdem & Eminoğlu-Küçüktepe, 2024; Keskin et al., 2024; Orhan, 2025; Seçer, 2021). In these studies, teachers reported difficulties such as a lack of materials and resources, limited technological infrastructure, limited lesson-planning time, and the need for additional time for HEM implementation. Furthermore, studies on the same subject have indicated that deficiencies in physical infrastructure and equipment negatively affect teaching processes (Seçer, 2021). It can be stated that both infrastructure problems related to the general education system and implementation difficulties arising from HEM being a relatively new model in Türkiye have contributed to these challenges. The lack of educational technologies and appropriate equipment may limit the full realization of the model's targeted learning outcomes. Therefore, teacher training, resource provision, and strengthening school infrastructure are critical for the sustainable and effective implementation of HEM (Keskin et al., 2024).

In the recommendations category, based on teacher opinions, it was determined that for HEM to be implemented sustainably and effectively, teacher training should be increased, alignment with the curriculum should be ensured, sufficient resource support should be provided, successful application examples should be shared, and mentoring systems should be developed. Similar results have been reported in studies on this subject (Erdem & Eminoğlu-Küçüktepe, 2024; Keskin et al., 2024; Koçoğlu, 2018; Seçer, 2021). In these studies, teachers emphasized the importance of increasing training, providing resources and technological infrastructure, supporting teachers in implementation, and integrating the model into the formal education curriculum for effective HEM implementation. In the study conducted by Ayra et al. (2025), teachers also stated that it is necessary to increase the number of lessons implemented, establish specialized HEM classes, provide teachers with guidance materials, and provide technological support. The planning, implementation, and technological equipment required by the model can create difficulties if teachers do not receive adequate training and support. Furthermore, infrastructure differences between schools and large class sizes are prominent factors limiting the implementation of the recommended practices. Therefore, recommendations for implementation, such as increasing teacher training, developing mentoring systems, and providing resources and technological infrastructure, can be considered critical factors that support the sustainability of the model and the development of students' problem-solving skills (Ayra et al., 2025; Ceylan et al., 2020).

This study found that HEM-supported mathematics instruction enhances students' ability to develop strategies, as well as their analytical and creative thinking skills in the problem-solving process; it also supports collaborative learning and motivation by increasing classroom interaction. This finding

demonstrates that HEM is a flexible, student-centered teaching approach that holistically develops cognitive and social-emotional dimensions. These results are consistent with studies in the literature on the subject (Ceylan et al., 2020; Çimşir, 2024; Erdem & Eminoğlu-Küçüktepe, 2024; Keskin et al., 2024; Koçoğlu, 2018). In these studies, teachers reported that HEM increased students' active participation in class, supported creativity and productivity, encouraged collaboration and teamwork, and supported meaningful learning through the use of technological tools and equipment (Erdem & Eminoğlu-Küçüktepe, 2024; Zeybek et al., 2024). Similarly, Seçer (2021) and Orhan (2025) observed in their studies that collaborative learning strengthens students' motivation, self-confidence, and collaboration skills in the social-emotional dimension. These findings are consistent with the theoretical framework of collaborative learning. While the model aims to holistically develop students' cognitive and social-emotional skills, it also supports 21st-century skills such as collaborative learning, creativity, problem solving, and motivation (Babayiğit-Durakcan, 2024; Ceylan et al., 2020). Thus, HEM can be considered a flexible, student-centered teaching approach that enhances students' academic achievement and social interactions (Ayra et al., 2025; Keskin et al., 2024).

The quantitative findings showed that the control and experimental groups were equivalent at the pretest; however, after the intervention, the experimental group demonstrated significantly higher MPS and CT scores than the control group. Significant differences were observed in favor of the experimental group in the self-assessment, problem-solving knowledge, and metacognition subdimensions of problem-solving, as well as in the skepticism, curiosity, open-mindedness, and biasness dimensions of CT. The qualitative findings supported these results by revealing that, due to the interdisciplinary structure of the HEM, students engaged in collaborative, inquiry-based, and strategy-oriented work on real-world related problems. Teacher perspectives indicated that these learning environments enabled students to become aware of their thinking processes and to evaluate different perspectives, thereby explaining the cognitive improvements observed in the quantitative findings. Nevertheless, limitations related to material and time constraints, as well as teacher competencies, emerged as factors affecting the model's effectiveness. Overall, when the quantitative and qualitative findings are considered together, the results suggest that HEM is an effective interdisciplinary model for supporting problem-solving and CT skills in mathematics education; however, pedagogical and structural support is required to ensure its sustainability.

Limitations and Implications

Notwithstanding its contributions, this study has certain limitations that should be taken into account when interpreting the results. Firstly, the research was carried out with a relatively small sample drawn from a specific regional context, which may limit the extent to which the findings can be generalized. Second, the quasi-experimental design without random assignment may limit the ability to establish strong causal inferences. Third, the data collection process relied on self-report scales, which may be subject to response bias. In addition, the qualitative findings were based on a limited number of teachers and may therefore not fully reflect diverse perspectives across different educational settings.

Despite these limitations, the findings provide important implications for both practice and research. From a practical perspective, the results suggest that integrating HEM into primary school mathematics instruction can support the development of students' MPS and CT skills. The interdisciplinary and technology-supported structure of HEM offers opportunities to design more engaging and meaningful learning environments. In this context, supporting teachers through professional development programs, improving access to instructional materials, and strengthening technological infrastructure may enhance the effectiveness and sustainability of HEM-based practices.

From a research perspective, future studies could investigate the long-term effects of HEM using larger and more diverse samples and true experimental designs. Additionally, further research may explore the impact of HEM on other variables such as students' motivation, attitudes, and

collaborative learning skills. Examining the implementation of HEM across grade levels and subject areas would also deepen understanding of its interdisciplinary potential.

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Ethics and Conflict of Interest

The study was carried out in accordance with the principles of the Declaration of Helsinki and the ethical standards set by the Turkish Council of Higher Education. Ethical approval was granted by the Ethics Committee of Bayburt University on May 07, 2025 (Decision No: 195; Session No: 5), covering both the implementation and data collection phases. Participants' rights and confidentiality were protected throughout the study. Data were collected solely for scientific purposes, analyzed anonymously, and handled in line with institutional regulations. The authors declare that they have adhered to ethical standards at all stages and declare no conflict of interest.

Author Contributions

Tunahan Filiz contributed to conceptualization, methodology, conducting the interviews, data analysis and interpretation, and writing the original draft. Bilgen Duran Erel contributed to writing—original draft, investigation, formal analysis, data curation, resources, and conceptualization.

Data availability

The data that support the findings of this study are available on request from the corresponding author.

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