IOJPE

ISSN: 1300 – 915X
www.iojpe.org

International Online Journal of Primary Education *2017, volume 6, issue 1*

EXAMINATION OF THE PEDAGOGICAL CONTENT KNOWLEDGE OF MATHEMATICS TEACHERS

Assist. Prof. Dr. Burçin GÖKKURT ÖZDEMİR
Department of Mathematics and Science Education, Faculty of Education, Bartın University, Bartın, Turkey
gokkurtburcin@gmail.com

Prof. Dr. Yasin SOYLU
Department of Mathematics and Science Education, Faculty of Kazım Karabekir Education, Atatürk University, Erzurum, Turkey
yasinsoylu@gmail.com

Abstract
Mathematics education consists of numerous factors, and a vital factor is teachers. Therefore, well-trained teachers play a crucial role in the education system. In this context, the pedagogical content knowledge of teachers forms a vital foundation for effective mathematics teaching. The study aimed to examine the pedagogical content knowledge of mathematics teachers. Forty-one mathematics teachers took part in this study. The qualitative research method was applied and eight open-ended questions were asked for data gathering purposes. The content and descriptive analysis methods were used for data analysis. The study showed that the instructional explanations of teachers were typically at their instrumental level (content-level). Accordingly, it can be conferred that teachers lack the sufficient level of conceptual knowledge required for the effective mathematics teaching that is the aim of the curriculum. Therefore, it was suggested that teachers develop their pedagogical content knowledge through in-service courses in line with curriculum targets.
Keywords: Instructional explanations, mathematics teaching, pedagogical content knowledge

INTRODUCTION

In today's age of information, considerable changes are taking place within mathematics education in terms of what mathematics is and how it should be taught (The National Council of Teachers of Mathematics [NCTM], 2000). With the changing perspective in education, a shift has occurredfrom conveying the existing knowledge to showing ways to reach educational knowledge in teacher-student relationships. The objective is to teach individuals with the ability to solve problems and use their acquired knowledge in everyday life. Mathematics classes are important in teaching these skills (Baki, 2006). Kaptan and Kuşakçı (2002) asserted that effective mathematics education aims to impart skills in scientific and rationalist thinking and raise creative and productive individuals who have the means to find, use and share knowledge, rather than simply memorising it.

Teachers and students are among the most important factors in making this effective mathematics education possible (Toluk-Uçar, Pişkin, Akdoğan, &Taşçı, 2010). Therefore, well-trained teachers are important in an effective education system (Türnüklü & Yeşildere, 2007; Yüksel, 2008) and they need to know mathematical knowledge for teaching (Kazima, Pillay, & Adler, 2008). This is because students construct mathematics knowledge with teachers through their own experiences. A student's understanding of mathematics is shaped by the teaching that the student receives at school (Aksu, Demir, & Sümer, 1998). For this reason, the quality of education directly depends on teacher education, which has gained further importance (Karal-Eyüboğlu, 2011; Karal & Alev, 2016).

Teacher education, the importance of the teacher, the role of the teacher, and the qualities that a teacher needs to possess are contemporary topics and concepts (Baskan, 2001). Many education reforms have been made in developed countries and many studies have, in recent years, directed their focused interest on teachers in terms of the training and qualities they need to posses (Bolat & Sözen, 2009; Meriç & Tezcan, 2005). In particular, studies on the pedagogical content knowledge of teachers have provided people around the world with alternative view points on teacher education (Lesniak, 2003). In this regard this study is expected to be beneficial for South African Teachers. Finally, it is also the case that enhancing the efficiency of the teacher education programmes (pre-service and post-service programmes), which is crucial for the creation and development of teachers' knowledge base, will only be possible with the use of research findings to be obtained in this field. However, several studies show that prospective teachers and teachers graduating from faculties of education experience certain problems in conveying pedagogical knowledge to students. (Canbazoğlu, 2008; Dani, 2004; Halim&Meerah, 2002; Feiman-Nemser& Parker, 1990; Tirosh, 2000; Toluk-Uçar, 2009). Similarly, Ball (1990a, 1990b) stated in his studies that prospective teachers' pre-and post-university understanding of mathematics is insufficient for elementary education. Although prospective teachers generally understand what the rules and methods are and how they are applied, they are not able to find explanations for the underlying meanings of given situations. Another study put forward that mathematics teachers have difficulty in acquiring mathematical content knowledge (Yüksel, 2008). Teachers should have accurate conceptual and relational knowledge, and they must be able to explain the underlying meanings and principles for an effective mathematics education (Ball, 1990a). Therefore, prospective mathematics teachers should possess pedagogical knowledge along with the subject knowledge to become good teachers (Ball, 1990b).

Ever since Shulman (1986) described pedagogical content knowledge as the keystone of his Knowledge Growth in Teaching Project, teacher educators have given rapidly more attention to the study of the learning-to-teach process (Stengel & Tom, 1996). Shulman (1986, 1987) expressed pedagogical content knowledge as "that special combination of content and pedagogy". In other words, pedagogical content knowledge functions as a bridge between field knowledge and pedagogical knowledge. According to Shulman (1997), one important aspect of pedagogical content knowledge is that it hones disciplined thinking skills in students and helps them in comprehending concepts (cited in Monte-Sano, 2011). Shulman (1986) states that teachers possessing pedagogical content knowledge should also have the following skills, knowledge of the most functional representation of subjects and concepts, knowledge of what facilitates and can complicate the learning process, knowledge of students' misconceptions, knowledge of simulations, representations, examples and explanations to clarify concepts and remove misconceptions, and knowledge of the ideas, perceptions and preliminary knowledge that students possess for course subjects at different ages and levels. According to An, Kulm, and Wu (2004), pedagogical content knowledge consists of three fundamental components: content knowledge, curriculum knowledge and teaching knowledge.

Subject field knowledge is the basic conceptual and contextual knowledge of teachers about (mathematics, biology, chemistry etc.) their field (Uşak, 2005). Various studies have proved that teachers commonly lack subject field knowledge. Teachers lacking in subject field knowledge typically define concepts and relations incompletely. Such teachers follow a teacher-centred instruction approach. In addition, they create learning environments where students' questions are ignored, and no healthy student participation takes place (Kılcan -Arslan, 2006). Teachers are having difficulty with both subject field knowledge and field-specific pedagogical knowledge. Teachers particularly experience difficulty in providing good educational explanations for mathematical rules and concepts which are among the most important aspects of mathematical-specific pedagogical knowledge. This is largely due to the fact that teachers' educational explanations are mainly based on memorisation, rules and practices (Kinach, 2002a, 2002b). Due to all these inadequacies, both students and teachers continue their teaching process with imperfect knowledge.

According to Batura, and Nason (1996), a correct solution in mathematics does not show that learning took place. In mathematics, learning a subject means developing solutions, and knowing why a certain calculation method functions, gives the correct solution and how different concepts are related, because questioning lies at the heart of mathematics. Türnüklü and Yeşildere (2007) states that the mathematics teaching knowledge of the teacher shows how good he/she is, rather than how much mathematics knowledge he/she has. For this reason, the pedagogical content knowledge of teachers should be evaluated. Various levels of understanding are developed to evaluate teachers' pedagogical content knowledge. Kinach's levels of mathematical understanding are among these (2002a, 2002b).

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ISSN: 1300 – 915X
www.iojpe.org

International Online Journal of Primary Education 2017, volume 6, issue 1

Kinach (2002a, 2002b) focuses on Perkins and Simmons's (1988) levels of mathematical understanding and groups them under two headings: instrumental understanding and relational understanding. Instrumental understanding contains *what* and *how* knowledge, while relational understanding shows the reasons underlying *what* and *how*. Within this perspective, instrumental understanding is handled within the scope of *context level*, which (aims to explain individuals, rules and practices superficially). Relational understanding is formed of four levels of understanding. These are *the concept level*, which (uses qualities of concept and different meanings of concept, and includes identifying patterns and relationships and categorizing, into a class, the phenomena possessing them), *the problem-solving level*, which (uses analytic methods such as induction, deductive thinking, special problem-solving techniques and mathematical modelling (which also includes metacognitive and subject-specific strategies, and guiding schemes), *the epistemic level*, which (contains information about the knowledge itself, i.e. the source of knowledge (According to Perkins and Simmons (1988), this level expresses the fact underlying the explanations, and puts forward reasons for thinking and concept and problem-solving levels) and *the inquiry level* which is (advanced problem-solving level in which new knowledge, and different problems or theorems are suggested). Previous studies mainly examined the pedagogical content knowledge of pre-service teachers (Baştürk & Dönmez, 2011; Bukova-Güzel, Cantürk-Günhan, Kula, Özgür& Elçi, 2013; Gökbulut, 2010; Şahin, et al., 2013; Toluk-Uçar, 2011). There are many studies (Dani, 2004; Halim &Meerah, 2002; Karal-Eyüboğlu, 2011; Lee &Luft, 2008; Özel, 2012) on the science teachers. In addition, most of studies mainly focused on the content knowledge of teachers and prospectiveteachers(Moats & Foorman, 2003; Van der Sandt & Nieuwoudt, 2003). In this context, it is important to know the pedagogical content knowledge of mathematics teachers. Therefore, the main purpose of thisstudy is toidentify the level of explanations given by mathematics teachers to mathematical situations. For this reason, I aim to contribute to filling this gap in the literature.

METHODOLOGY

The case study method which is based on the qualitative research approach has been used in this study. A case study examines a bounded system, or a case, over time in depth, employing, multiple sources of data found in the setting (Mcmillan & Schumacher, 2010). Therefore, the case study technique was used in our study in order to thoroughly evaluate the pedagogical content knowledge of mathematics teachers.

Participants

The participants of this study consisted of forty-one secondary mathematics teachers. Teachers were selected by the purposive sampling method. The teachers were selected on the basis of their willingness to participate in the study. Teachers' names were coded such as T_1,T_2, T_3,T_4 etc.

Data Collection Tool

Ten open-ended questions were used as the data gathering tool with a view to increase its validity in line with the literature (Ball, 1990b; Kinach, 2002a, 2002b; Pesen, 2006; Toluk-Uçar, 2011). Two field experts were asked whether the given mathematical expressions could reveal pedagogical content knowledge and after making some adjustments, two questions were left out. One reason for leaving out these questions was that there was another question measuring the same pedagogical content knowledge, and the other reason was that one question did not have the attributes required to measure the pedagogical content knowledge of the teachers. The data gathering tool thereby took its final shape with eight open-ended questions. The data were gathered by research through semi-structuredface-to-faceinterviews and teachers' written responses. The interviews were designed to explore participants' ideas, feelings, and understandings about some mathematics subjects. Participants were asked to write in detail how they explain given mathematical situations to someone learning it for the first time and how they felt while they were answering the questions. The first and second questions were related to fractions, the third and fourth questions were about exponential numbers, and the fifth question asked whether the number zero is odd or even. The sixth question indicated why a positive result is achieved after multiplying two negative numbers, the seventh question stated why a zero factorial is equal to one and the eighth question was about the solution set of a number whose square root is nine. These questions are presented below:

1. $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$
2. $\frac{1}{4} : \frac{1}{2} = \frac{1}{2}$
3. $a^n . a^m = a^{n+m}$
4. $a \neq 0,\ a^0 = 1$
5. whether the zero number is odd or even
6. (-3).(-4)=(+12)
7. 0!=1
8. $x^2 = 9,\ \ x = 3\ or\ x = -3$

Data Analysis

Kinach's mathematical understanding level was used as the framework of the data analysis. Within the framework of this mathematical understanding level, a teacher can make explanations at many different levels of understanding. These levels were coded as A: Content level, B: Concept level, C: Problem-solving level, D: Epistemic level and E: Inquiry level. After examining the answers given by teachers, the researchers sub-coded the levels of understanding as follows:

A Content level *A1: Superficial explanation of rules or given expressions* *A2: Misuse of visual elements* *A3: Using analogies* *A4: Solving equations by giving values* *A5: Using meaningless expressions*
B Conceptlevel *B1: Correct use of visual elements* *B2: Explaining by sampling* *B3:Creating patterns* *B4:Using concept features and different meanings*
C Problem-solving level *C1: Developing concept-specific strategies with number or fraction problems*
D Epistemiclevel *D1:Correct use of visual elements and showing their grounds* *D2: Analysis method and reasons for using it* *D3: Using definitions and features and giving justifications* *D4:Taking square root or creating equations and giving justifications* *D5: Using permutation and giving reasons for using it*
E Inquirylevel *E1:Putting forward new knowledge*

The qualitative data analysis methods were applied as content and descriptive analysis methods. It is necessary to make a reliability study in cases where multiple researchers collaborate on data analysis. In such a case, researchers code the same data set and make a numerical comparison of coding similarities and differences to reach a coding percentage. A reliability level of 70% is required in such studies (Yıldırım & Şimşek, 2013). Therefore, after an evaluation of the study data, two researchers performed coding at different times. Subsequently, the codes were compared and the coding reliability level was found to be 82%. The remaining 18% was corrected after the researchers reached a

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ISSN: 1300 – 915X
www.iojpe.org
International Online Journal of Primary Education 2017, volume 6, issue 1

consensus. Later, the data was classified under specified codes and rendered meaningful for readers, and the unnecessary codes were left out, keeping to the aim of the study.

RESULTS

The findings were gathered from interviews and written responses of mathematics teachers given in eight open-ended questions. Accordingly, below are the findings showing how teachers use their pedagogical content knowledge.

Table 1. *Teachers' Answers to the First Question*

A	Content level	f
A1	Superficial explanation of rules or given expressions	28
A2	Misuse of visual elements	5
A5	Using meaningless expressions	3
B	**Concept level**	
B1	Correct use of visual elements	9
B2	Explaining by sampling	3
C	**Problem- solving level**	
C1	Developing concept-specific strategies with number or fraction problems	1
D	**Epistemic Düzey**	
D1	Correct use of visual elements and showing their grounds	2
E	**Inquiry level**	
E1	Putting forward new knowledge	0

Table 1 indicates that teachers' instructional explanations for the first question, levels of understanding and the code frequencies of these levels. According to mathematics teachers' answers to the first question, mostly code A explanations were used. Some of the teachers at this level superficially explained how the rule will be applied, and some tried to use visual elements. In addition to this, some teachers used meaningless expressions. Conversely, very few teachers used explanations at codes B and D.

Teachers' explanations show that most are not knowledgeable about the conceptual knowledge underlying subtraction operation in fractions, and they only used relational knowledge. Teachers' instructional explanations also support this idea.

T_3 : "First, I will explain that it's a rational operation, and denominators are equalised in summation and subtraction of rational numbers."
T_{20}: "Denominators are made equal. This is taught as a rule."

Some teachers who tend to misuse visual elements were mistaken in the subtraction process of fractions by subtracting a partition different to the partition from the initial whole. Teacher T_{31}'s answer is presented below:

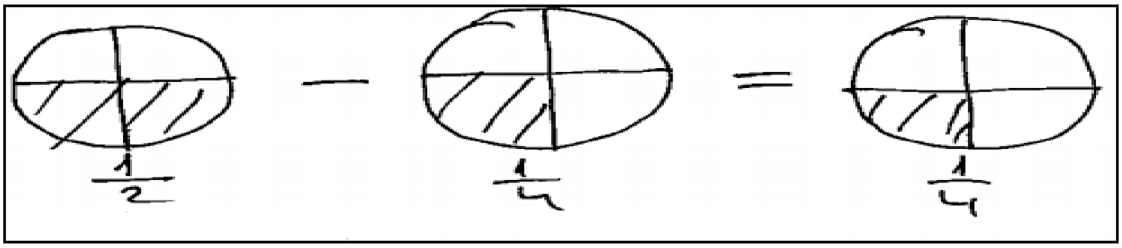

Figure 1 T_{31}teacher's answer to question no. 1

Table 2 shows that teachers' instructional explanations for the second question, levels of understanding and the code frequencies of these levels.

Table 2. *Teachers' Answers to the Second Question*

A	**Content level**	**f**
A1	Superficial explanation of rules or given expressions	34
A2	Misuse of visual elements	2
B	**Concept level**	
B1	Correct use of visual elements	4
B3	Creating patterns	1
C	**Problem-solving level**	
C1	Developing concept-specific strategies with number or fraction problems	1
D	**Epistemic level**	
D1	Correct use of visual elements and showing their grounds	2
D2	Analysis method and reasons for using it	1
E	**Inquiry level**	
E1	Putting forward new knowledge	0

In Table 2, the majority of teachers proposed code explanations for the second question. Very few educational explanations were given in codes B, C and D. The majority of the teachers stated that they teach division operation in fractions as a rule, and this shows that they have code A pedagogical content knowledge. Quotes from some of the teachers are presented below:

T_9: "I teach division operation as the opposite of multiplication operation."
T_{15}: "After explaining how division takes places in fractions (rational numbers), I continue with telling them to write the first fraction as it is, and reversing the second fraction and multiplying them."
Table2 shows that teachers are insufficient in explaining the division operation in fractions and most illustrate (A1) this superficially.

Table 3. Shows that teachers' instructional explanations for the third question, levels of understanding and the code frequencies of these levels.

Table 3. *Teachers' answers to the Third Question*

A	**Content level**	**f**
A1	Superficial explanation of rules or given expressions	20
B	**Concept level**	
B2	Explaining by sampling	9
C	**Problem-solving level**	
C1	Developing concept-specific strategies with number or fraction problems	0
D	**Epistemic level**	
D3	Using definitions and features and giving justifications	14
E	**Inquiry level**	
E1	Putting forward new knowledge	0

In Table 3, it is shown that teachers mostly answered in codes A and D for the third question. No answerswere given in codes C and E. We can infer that teachers' pedagogical knowledge about the multiplication operation with exponential numbers (D3) with the same base is better than that of

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fractions. All teachers answering in code D used the definition of exponential numbers and tried to give a logical justification behind why exponents should be added while multiplying exponential numbers with the same base. Teacher T_{31}'s answer to this is presented below:

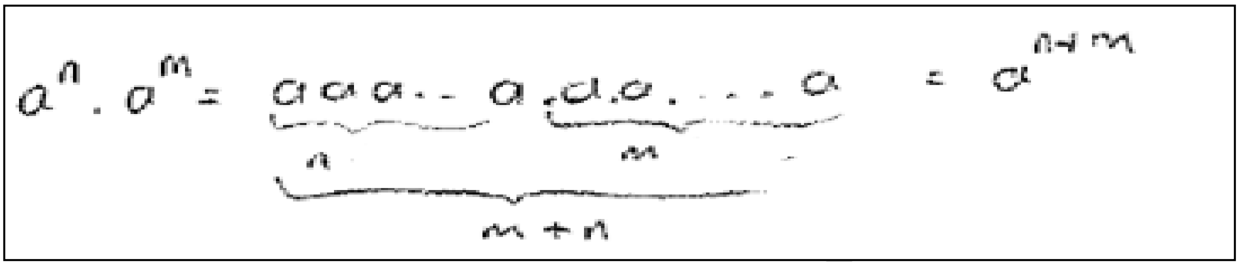

Figure 2 T_{31} teacher's answers to question no. 3

Table 4 shows that the teachers' instructional explanations for the fourth question, their levels of understanding and the code frequencies of these levels.

Table 4. *Teachers' Answers to the Fourth Question*

A	**Content level**	**f**
A1	Superficial explanation of rules or given expressions	18
A5	Using meaningless expressions	7
B	**Concept level**	
B2	Explaining by sampling	2
B3	Creating patterns	7
C	**Problem-solving level**	
C1	Developing concept-specific strategies with number or fraction problems	0
D	**Epistemic level**	
D3	Using definitions and features and giving justifications	7
E	**Inquiry level**	
E1	Putting forward new knowledge	0

Table 4 shows that most of the teachers gave code A answers for the fourth question. No answers were given in codes C and E. Some of the teachers taught as a rule that exponents of all numbers apart from zero were equal to zero, while some provided meaningless expressions (A5) to clarify it. Related teachers' answers are presented below:

T_{12}: Since "a" is different from zero, we give the smallest value.
T_{20}: When the exponent of numbers apart from zero is 0, the answer is 1. This is taught as a rule.
In contrast, teachers answering in code B tried to explain the mathematical expression $a^0=1$ with more patterns (B3). Teacher T_{29}'s answer is presented below:

Figure 3 T_{29} teacher's answers to question no. 4

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ISSN: 1300 – 915X
www.iojpe.org
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In Table 4, we can clearly see that some of the teachers gave explanations in code D and solved the problem by only using level D3, which only provides the logic behind the question by using the properties of exponential numbers. Teacher T_{11}'s written statement below clearly indicates the motive behind the question.

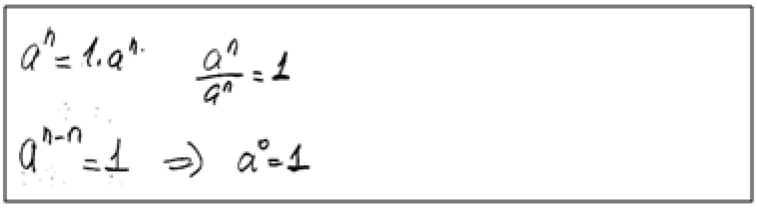

Figure 4 T_{11} teacher's answers to question no. 4

Table 5 summarizes the teachers' instructional explanations for the fifth question, their levels of understanding and the code frequencies of these levels.

Table 5.*Teachers' Answers to the Fifth Question*

A	**Content level**	**f**
A1	Superficial explanation of rules or given expressions	13
A5	Using meaningless expressions	12
B	**Concept level**	
B2	Explaining by sampling	1
B4	Using concept features and different meanings	11
C	**Problem-solving level**	
C1	Developing concept-specific strategies with number or fraction problems	0
D	**Epistemic level**	
D3	Using definitions and features and giving justifications	6
E	**Inquiry level**	
E1	Putting forward new knowledge	0

Table 5 clearly summarizes that most of the teachers do not have sufficient knowledge about whether the number zero is even or not. The majority of the teachers provided educational explanations in codes A and B. Two teachers using code A_5 in particular could not identify that the number zero is even, and stated that zero is neither odd nor even. Their answers were as follows:

T_{18}: "Zero means null. Thus, we can't say whether it is odd or even."

T_{22}: "Zero means null. We can't state the oddness or evenness of a non-existing expression."

No answers are given in codes C and E, and only six teachers provided explanations in code D. Teachers in this level emphasized why zero is even and gave the logic behind it. A good example is teacher T_{14}'sanswer:

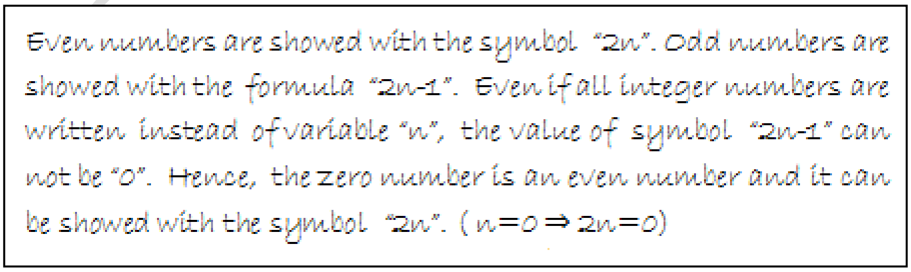

Figure 5 T_{14} teacher's answers to question no. 5

Table 6 summarizes the teachers' instructional explanations for the sixth question, their levels of understanding and the code frequencies of these levels.

Table 6. *Teachers' Answers to the Sixth Question*

A	Content level	f
A1	Superficial explanation of rules or given expressions	28
A3	Using analogies	5
B	**Concept level**	
B1	Correct use of visual elements	1
B4	Using concept features and different meanings	3
C	**Problem-solving level**	
C1	Developing concept-specific strategies with number or fraction problems	2
D	**Epistemic level**	
D1	Correct use of visual elements and showing their grounds	4
E	**Inquiry level**	
E1	Putting forward new knowledge	0

We can infer from Table 6 that more than half of the teachers answered in level A1 to the question that a positive integer is found when we multiply two negative integers, and very few answers are given in levels B, C and D. In addition, it is striking that two of the teachers (T_{16}, T_{33}) did not even reply to the question. Quotes below show how teachers are lacking in pedagogical knowledge.

T_{1}: *I teach it as a rule that when you multiply two integers with the same sign, you get a positive result; and when you multiply two numbers with opposite signs, you get a negative result. I don't hand out counting tokens.*

T_{13}: *First I state that integers can be multiplied, multiplication of the same sign provides a positive result, and multiplication of opposite signs provides a negative result, then I tell them to first multiply signs, then the numbers.*

Written statements of the teachers show that they make use of analogies (A2) when clarifying that the multiplication of two negative numbers results in a positive number. However, there is no explanation as to why the result is positive. Some quotes from teacherT_9 and T_{15} are presented below:

T_9: *I use the principle "the enemy of my enemy is my friend".*

T_{15}: *I use the usual statements of "my friend's friend", "my friend's enemy" and "my enemy's enemy".*

Table 7 shows that the teachers' instructional explanations for the seventh question, their levels of understanding and the code frequencies of these levels.

Table7. Teachers' Answers to the Seventh Question

A	Content level	f
A1	Superficial explanation of rules or given expressions	19
A5	Using meaningless expressions	9
B	**Concept level**	
B2	Explaining by sampling	1
C	**Problem-solving level**	
C1	Developing concept-specific strategies with number or fraction problems	0
D	**Epistemic level**	
D3	Using definitions and features and giving justifications	4
D5	Using permutation and giving reasons for using it	2
E	**Inquiry level**	
E1	Putting forward new knowledge	0

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ISSN: 1300 – 915X
www.iojpe.org
International Online Journal of Primary Education
2017, volume 6, issue 1

When we look at Table 7, we can see that most teachers answered in code A like in previous questions and 0!=1 is considered to be a special rule. Some of the teachers tried to base this rule on illogical grounds. Here are their answers:

$T_{6:}$ *I clarify that 0!=1 is specifically used to show that the result is not 0 in multiplication of consecutive numbers.*

$T_{13:}$ *A positive integer's factorial is the multiplication of all positive integers smaller than itself. This definition does not apply to 0!, because, there is no natural number smaller than zero. 0! is defined as 1 in line with the purpose of definition.*

Four of the mathematics teachers (T_4,T_{12},T_{16},T_{17}) gave no educational explanation for this question. Teachers answering in level B tried to explain 0!=1 with examples. Teachers answering in level D clarified why zero factorial equals one with justifications. They typically used the definition of factorial (D3) for this purpose. Here you can find teacher T_{33}'s answer:

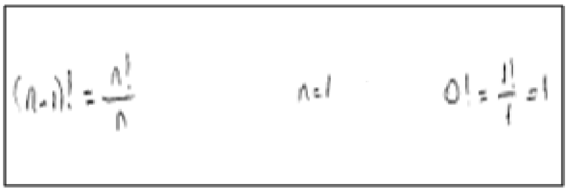

Figure 6 T_{33} teacher's answers to question no. 7

Table 8. *Teachers' Answers to the Eighth Question*

A	Content level	f
A1	Superficial explanation of rules or given expressions	19
A4	Solving equations by giving values	12
B	**Concept level**	
B1	Correct use of visual elements	3
B2	Explaining by sampling	1
C	**Problem-solving level**	
C1	Developing concept-specific strategies with number or fraction problems	0
D	**Epistemic level**	
D4	Taking square root or creating equations and giving justifications	8
E	**Inquiry level**	
E1	Putting forward new knowledge	0

Table 8 shows that the teachers' instructional explanations for the eighth question, their levels of understanding and the code frequencies of these levels.

Written statements from of the teachers tell us that there are teachers answering in codes A, B and D. Teachers using code A taught their students that the solution set of $x^2=9$ is (-3) and (+3), and superficially wrote them in their equational places. Some answers from teachers were given:

$T_{6:}$*Since the exponent is even in $x^2=9$, there will be 2 cases. I will tell students about (+) and (–).*
$T_{7:}$*Since (-3).(-3)=9 and (+3).(+3)=9, the solution set is (-3) and (+3).*

Considering the instructional explanations of the teachers answering in code B, they are clearly found to use visual elements (B1) or give examples (B2). The answer of a teacher in B1code is exactly cited below. Conversely, teachers in code D used the square root technique or equations to explain why the solution set of $x^2=9$ is (-3) and (+3).

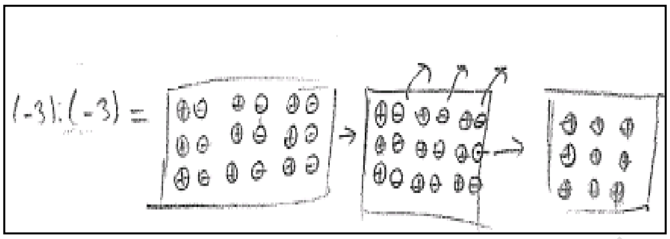

Figure 7 T_{34} teacher's answers to question no. 8

DISCUSSION, CONCLUSION and RECOMMENDATIONS

This study examined how mathematics teachers explain some mathematical statements about numbers to students. It was found that the teachers use their pedagogical content knowledge through different representation methods (visual elements, examples, analogies, equations etc.) As it is understood from the results of this study, the teachers' pedagogical content knowledge was not limited to certain representations. This was in parallel with the study result concluding that "teachers can display their pedagogical content knowledge with verbal expressions, numeric examples, analogies and thought-wise relations (Grouws & Schultz, 2004)."

It was concluded that the teachers' knowledge about division and subtraction in fractions is at subject level to a great extent, and they have misconceptions about subtraction and divisions in fractions. This result showed parallelism with studies such as those of (Azim, 1995; Ball, 1990a, 1990b; Borko et al., 1992; Cluff, 2005; Işıksal, 2006; Lubinski, Fox& Thomason, 1998; Ma, 1999; Nagle & McCoy, 1999; Simon & Blume, 1994). Althoughthe teachers have knowledge of division and subtraction rules, they are not aware of the underlying reasons. Considering the teachers' answers to questions about exponential numbers, factorials, the abnormality of the number zero, integers and solution sets of equations, their knowledge is predominantly at the subject level just, as in division and subtraction in fractions. Few teachers provided answers at concept, problem-solving and epistemic levels. A very striking result is that none of the teachers provided an answer at research level. These findings were supported by similar studies (Ball, 1990a, 1990b; Toluk-Uçar, 2011). Accordingly, we can say that teachers typically don't have sufficient pedagogical content knowledge, and therefore, teach mathematics based on memorisation. Similarly, Henningsen and Stein (1997), stated in their study that teachers provide educational explanations based on memorisation rather than understanding.

Another conclusion of the study is that the teachers do not have the sufficient conceptual level required by the curriculum for mathematics teaching purposes. This conclusion was in parallel with the results of studies by (Even, 1993; Gökkurt, 2014; Gökkurt & Soylu, 2016a, 2016b; Ma, 1999; Toluk-Uçar, 2011). However, teachers should have exceptional field knowledge (Ball, 1990a) and pedagogical content knowledge (An, Kulm & Wu, 2004; Borko et al, 1992; McDiarmid, Ball,& Anderson, 1989) for effective mathematics education, because, pedagogical content knowledge plays a significant role in developing the understanding of students, and students construct mathematics with teachers through their own experiences(Aksu, Demir,&Sümer, 1998).A student's understanding of mathematical education is shaped by the teaching they receive at school. This is why teachers should have pedagogical content knowledge at the conceptual level and have a conceptual level of understanding in mathematics. Therefore, it is possible to improve the pedagogical content knowledge of teachers via in-service courses in line with curricular objectives. This study was performed to evaluate the pedagogical content knowledge level of mathematics teachers. Similar studies can be conducted on teachers in other fields.

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